**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

Since the work begins 10 minutes after the car dropped. So time remain to do the servicing of the car is 50 min.

Now to calculate the probability for the service manager cannot meet his commitment is

P(T>50)= 1 – P(T<=50)

Here T is the time taken to complete the work.

Firstly we will find the z-score.

We have *Z*=(*X*−*μ*​)/*σ*

*=* (X-45)/8

Now P(T>50) =1 - P(T<=50) = P(Z <=(50-45)/8 )= P(Z<= 0.625) =73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

So the option B is correct.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

1. Mean = 38, SD = 6

*Z* =(*X*−*μ)*​/*σ*  
Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

1. Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS : The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The probability of getting value between a and b is 0.99

The probability of getting value outside a and b is 1-0.99 = 0.01

Probability to the lower tail = 0.01/2 = -0.005

Probability to the upper tail= 0.01/2 = 0.005

Z- Score for 0.005 is 2.576

a = μ - 2.576σ

a = 100 - 2.576 \* 20 = 48.48

b = μ + 2.576σ

b = 100 + 2.576 \* 20 = 151.52

so option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year